Soft Gluon k_t-Resummation and the Froissart bound

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Abstract

We study soft gluon k_t-resummation and the relevance of InfraRed (IR) gluons for the energy dependence of total hadronic cross-sections. In our model, consistency with the Froissart bound is directly related to the ansatz that the IR behaviour of the QCD coupling constant follows an inverse power law.

Key words: Froissart bound, QCD, total cross-section, resummation
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1. Introduction

In this paper, we discuss soft gluon k_t-resummation in the InfraRed (IR) region, with the aim to connect it to the energy dependence of the total hadronic cross-section in the high energy region. We shall make use of a model [1] for total cross-sections, which incorporates in an eikonal formulation such QCD inputs as mini-jets and soft gluon k_t-resummation. This model has been successfully applied both to proton and photon processes: our aim here is to describe its physical content, without explicit reference to data fitting, and explain how the model incorporates a taming effect on the rapidly rising QCD cross-sections, thus inducing a more temperate rise of the total cross-section.

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The energy behaviour of the total hadronic cross-section has been the focus of both theoretical and experimental enquiries for a long time. Fits inspired by theoretical arguments have been the subject of many debates. A traditional Regge-Pomeron type fit [2] such as

$$\sigma_{total} = X s^{-\eta} + Y s^{\epsilon}, \tag{1}$$

with $\eta, \epsilon > 0$, presents the difficulty of not agreeing with the Froissart-Lukaszuk-Martin bound [3, 4, 5]

$$\sigma_{total} \leq \frac{\pi}{m_\pi^2} \ln^2(s/s_0). \tag{2}$$

However, with $\sqrt{s_0} \simeq \mathcal{O}(1\ GeV)$, the large constant factor at the r.h.s of Eq. (2) makes possible for Eq. (1) to be valid only in the present region, and the observed behaviour to be not yet asymptotic. Still, phenomenological reasons run against the validity of the Regge-Pomeron type expression with a universal value for ϵ, since LEP data on photon-photon indicate that the power, with which $\sigma_{total}^{\gamma\gamma}$ rises, differs from that in $pp/p\bar{p}$ [6].

Currently, many models focus on QCD perturbative processes to drive the rise with energy, with QCD inspired models on the one hand [7, 8] and approaches based on Reggeon calculus [9, 10] on the other. However, it is still being debated how to implement this dynamics and simultaneouly describe both the early rise, which starts for $\sqrt{s}$ between $10 \div 20\ GeV$ and $50 \div 60\ GeV$, and the subsequent levelling off at higher energies [11].

In this paper we shall show how both of the above features are present in our model for the total cross-section [1] and relate the rate, at which the cross-section asymptotically rises, to the IR limit of soft k_t-resummation, thus linking directly the rise of the total cross-section to the IR region of QCD. We start by recalling in Sect. 2 some features of resummation and present, in Sect. 3, our proposal for handling the ultra soft gluon emissions which affect scattering at the very large impact parameter values, relevant to total cross-section calculations. After a brief discussion of our model in Sect. 4, we show in Sect. 5 the connection between the Froissart bound and our proposed eikonalized mini-jet model with ultra soft gluons. Finally, in Sect. 6 we discuss both the various energy scales and the constants involved in the model. We find that resummation of soft gluons emitted with very small transverse momentum k_t introduces a new energy scale, which modifies the constant in front of the asymptotic limit derived in our model for the total cross-section.

2. Resummation and the IR limit

The high energy behaviour of the total cross-section depends on the properties of the scattering amplitude at large values of the impact parameter b in the plane perpendicular to the scattering. Recent attempts to study this large-b behaviour have focused on relating Yang-Mills theories to string theories through the AdS/CFT correspondance [12, 13, 14]. Our approach to the large b-limit of the impact picture in the eikonal representation [15] is of a more phenomenological nature: within such a picture, we exploit soft gluon k_t-resummation in the IR region to describe matter distribution inside the hadrons as they engage in hard scattering and their parton constituents "see" each other. We have already supplied phenomenological evidence for the applicability of our model to high energy scattering [1]. Here we discuss some features of soft gluon k_t-resummation which bear on the asymptotic form of the total cross-section.

We start by recalling some properties of soft photon resummation. In QED, the general expression for soft photon resummation in the energy-momentum variable K_μ can be obtained order by order in perturbation theory [16, 17] as

$$d^4P(K) = d^4K \int \frac{d^4x}{(2\pi)^4} e^{iK\cdot x - h(x,E)} \tag{3}$$

where $d^4P(K)$ is the probability for an overall 4-momentum K_μ escaping detection,

$$h(x, E) = \int_0^E d^3\bar{n}(k)[1 - e^{-ik\cdot x}] \tag{4}$$

with $d^3\bar{n}(k)$ being the single soft photon differential spectrum, and E the maximum energy allowed for single photon emission.

Eq. (3) leads to the well known, power-like, form of the energy distribution [18]. This is not possible for the momentum distribution, but it is also not necessary, since the first order expression in α_{QED} is adequate. On the other hand, for strong interactions we require the resummed, transverse momentum distribution, namely

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \frac{1}{(2\pi)^2} \int d^2\mathbf{b}\ e^{-i\mathbf{K}_\perp\cdot\mathbf{b} - h(b,E)} \tag{5}$$

with

$$h(b, E) = \int d^3\bar{n}(k)[1 - e^{i\mathbf{k}_\perp\cdot\mathbf{b}}] \tag{6}$$

For large transverse momentum values, by neglecting the second term and using a constant cut-off as lower limit of integration, the above expression coincides with the Sudakov form factor [19].

Eq. (5) has been applied in QCD [20, 21, 22, 23, 24] with

$$h(b,E) = \frac{16}{3}\int^{E} \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\ln\frac{2E}{k_t}[1-J_0(k_t b)] \tag{7}$$

but its use is complicated by our ignorance of the IR behaviour of the theory. To overcome the difficulty arising from the IR region, the function $h(b,E)$, which describes the relative transverse momentum distribution induced by soft gluon emission from a pair of, initially collinear, colliding partons at LO, is split into

$$h(b,E) = c_0(\mu,b,E) + \Delta h(b,E), \tag{8}$$

where

$$\Delta h(b,E) = \frac{16}{3}\int_{\mu}^{E} \frac{\alpha_s(k_t^2)}{\pi}[1-J_o(bk_t)]\frac{dk_t}{k_t}\ln\frac{2E}{k_t}. \tag{9}$$

Since the integral in $\Delta h(b,E)$ now extends down to a scale $\mu \neq 0$, for $\mu > \Lambda_{QCD}$ one can use the asymptotic freedom expression for $\alpha_s(k_t^2)$. Furthermore, having excluded the zero momentum region from the integration, $J_o(bk_t)$ is assumed to oscillate to zero and neglected. The integrand in Eq. (9) is now independent of b and the integral can be performed. In the range $1/E < b < 1/\Lambda$, the effective $h_{eff}(b,E)$ is obtained by setting $\mu = 1/b$ [21]. This choice of the scale introduces a cut-off in impact parameter space which is stronger than any power, since the radiation function, for $N_f = 4$, is now [21]

$$e^{-h_{eff}(b,E)} = [\frac{\ln(1/b^2\Lambda^2)}{\ln(E^2/\Lambda^2)}]^{(16/25)\ln(E^2/\Lambda^2)} \tag{10}$$

The remaining b-dependent term, namely $exp[-c_0(\mu,b,E)]$, is dropped, a reasonable approximation if one assumes that there is no physical singularity in the range of integration $0 \leq k_t \leq 1/b$. This contribution however reappears as an energy independent smearing function which reproduces phenomenologically the effects of an intrinsic transverse momentum of partons. For most applications, this may be a good approximation. However, when the integration in impact parameter space extends to very large-b values, as is the case for the calculation of total cross-sections, the IR region may be important and the possibility of a physical singularity for α_s in the IR region becomes relevant. It is this possibility, which we exploit in studying scattering in the very large impact parameter region, $b \to \infty$.

3. A proposal for the IR limit in the soft gluon integral

In this Section we discuss a phenomenological expression for the coupling of ultra soft gluons to the emitting quarks, and compare the resulting large-b behaviour with the one discussed in the previous section.

Our choice for the IR behaviour of $\alpha_s(Q^2)$ used in obtaining a quantitative description of the distribution in Eq. (6), is inspired by the Richardson potential for quarkonium bound states [25], as we have proposed in a number of related applications [26]. Assume a confining potential (in momentum space) given by the one gluon exchange term

$$\tilde{V}(Q) = K(\frac{\alpha_s(Q^2)}{Q^2}), \tag{11}$$

where K is a constant, calculable from the asymptotic form of $\alpha_s(Q^2)$. Let us choose for $Q^2 << \Lambda^2$ the simple form

$$\alpha_s(Q^2) = \frac{B}{(Q^2/\Lambda^2)^p}, \tag{12}$$

(with B a constant), so that $\tilde{V}(Q)$ for small Q goes as

$$\tilde{V}(Q) \to Q^{-2(1+p)}. \tag{13}$$

For the potential, in coordinate space, $V(r) = \int d^3Q/(2\pi)^3 e^{i\mathbf{Q}\cdot\mathbf{r}}\tilde{V}(Q)$, Eq.(13) implies

$$V(r) \to (1/r)^3 \cdot r^{(2+2p)} \sim C\ r^{(2p-1)}, \tag{14}$$

for large r (C is another constant). A simple check is that for p equal to zero, the usual Coulomb potential is regained. Notice that for a potential rising with r, one needs $p > 1/2$. Thus, for $1/2 < p < 1$, this corresponds to a confining potential rising less than linearly with the interquark distance r, while a value of $p = 1$ coincides with the IR limit of the Richardson's potential and is also found in a number of applications to potential estimates of quarkonium properties [27].

Then, again following Richardson's argument, we connect our IR limit for $\alpha_s(Q^2)$ to the asymptotic freedom region using the phenomenological expression:

$$\alpha_s(k_t^2) = \frac{12\pi}{(33-2N_f)} \frac{p}{\ln[1+p(\frac{k_t^2}{\Lambda^2})^p]} \tag{15}$$

which coincides with the usual one-loop formula for values of $k_t >> \Lambda$, while going to a singular limit for small k_t, and generalizes Richardson's ansatz to values of $p \leq 1$. The range $p < 1$ has an important advantage, i.e., it allows the integration in Eq.(6) to converge for all values of $k_t = |k_\perp|$. Using Eq. (15), one can study the behaviour of $h(b, E)$ for the very large-b values which enter the total cross-section calculation and recover the perturbative calculation as well. The behaviour of $h(b, E)$ in various regions in b-space was discussed in [28], both for a singular and a frozen α_s, namely one whose IR limit is a constant. There we saw that, for the singular α_s case, the following is a good analytical approximation in the very large-b region:

$$\begin{aligned} h(b,M,) = {} & \frac{2c_F}{\pi}\left[\bar{b}\frac{b^2\Lambda^{2p}}{2}\int_0^{\frac{1}{b}}\frac{dk}{k^{2p-1}}\ln\frac{2M}{k} + 2\bar{b}\Lambda^{2p}\int_{\frac{1}{b}}^{N_p\Lambda}\frac{dk}{k^{2p+1}}\ln\frac{M}{k} + \bar{b}\int_{N_p\Lambda}^{M}\frac{dk}{k}\frac{\ln\frac{M}{k}}{\ln\frac{k}{\Lambda}}\right] \\ = {} & \frac{2c_F}{\pi}\left[\frac{\bar{b}}{8(1-p)}(b^2\Lambda^2)^p\left[2\ln(2Mb)+\frac{1}{1-p}\right] + \frac{\bar{b}}{2p}(b^2\Lambda^2)^p\left[2\ln(Mb)-\frac{1}{p}\right] + \right. \\ & \left. \frac{\bar{b}}{2pN_p^{2p}}\left[-2\ln\frac{M}{\Lambda N_p}+\frac{1}{p}\right] + \bar{b}\ln\frac{M}{\Lambda}\left[\ln\frac{\ln\frac{M}{\Lambda}}{\ln N_p} - 1 + \frac{\ln N_p}{\ln\frac{M}{\Lambda}}\right]\right] \end{aligned} \tag{16}$$

This approximation is valid in the region $b > 1/(N_p\Lambda) > 1/M$, with $N_p = (1/p)^{1/2p}$, $c_F = 4/3$ for emission from quark legs and $\bar{b} = 12\pi/(33-2N_f)$. The upper limit of integration, M, indicates the maximum allowed transverse momentum, to be determined by the kinematics of single gluon emission as in [29]. The above expression exhibits the sharp cut-off at large-b values which we shall exploit to study the very large energy behaviour of our model. On the other hand, the possibility that α_s becomes constant in the IR [21, 22, 23] in the same large b-limit leads to

$$h(b,M,\Lambda) = (constant)\ln(2Mb) + \ double \ \ logs \tag{17}$$

namely no sharp cut-off in impact parameter b, as expected.

4. The Bloch-Nordsieck(BN) model for the total cross-section

Our model for the total cross-section [1] is a modified mini-jet model, in which the rise with energy is driven by perturbative parton-parton scattering [30], tempered by an energy dependent acollinearity effect. This effect is due to k_t-resummation of soft gluon emission from the initial state, hereafter referred to as *soft gluon k_t-emission*. The emphasis on resummation, first introduced for electron scattering by Bloch and Nordsieck [31], gives the model its name. The model is built through the eikonal representation in impact parameter space, so as to satisfy unitarity, and allows to implement multiple parton scattering and to restore a finite size of the interaction through the impact parameter distribution in the scattering hadrons. The details of the model can be found in [1, 28, 32], here we shall recall some aspects relevant to its asymptotic energy behaviour.

In hadron-hadron scattering at a c.m. energy $\sqrt{s}$, unitarity allows to write a simple model for the total cross-section, namely

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\mathcal{I}m\chi(b,s)} cos\Re e\chi(b,s)] \approx 2\int d^2\mathbf{b}[1 - e^{-\bar{n}(b,s)/2}] \quad (18)$$

where the approximation on the r.h.s is obtained by neglecting the real part of the eikonal function (at the hadronic level, an acceptable approximation in the high energy limit) and $2\mathcal{I}m\chi(b,s) = \bar{n}(b,s)$. The latter follows from a semiclassical argument relating $\sigma_{inelastic}$ to a sum of a Poisson distributed independent, single and multiple collisions.

We use perturbative QCD to calculate the cross-section in order to obtain the average number of inelastic collisions. While implementing the QCD calculation, albeit approximate, we distinguish between the average number of collisions receiving contributions from hard physics processes and those from non perturbative ones, and write $\bar{n}(b,s)$ in the form

$$\bar{n}(b,s) = n_{NP}(b,s) + n_{hard}(b,s) \quad (19)$$

where the non perturbative (NP) term parametrizes the contribution of all those processes for which initial partons scatter with $p_t < p_{tmin}$, with p_{tmin} a suitable low energy cut-off for the QCD parton-parton cross-section. We parametrize $n_{NP}(b,s)$, which establishes the overall normalization, and focus our attention on the hard term, which is responsible for the high-energy rise and which we expect to dominate in the extremely high energy limit. We approximate this term as

$$n_{hard}(b,s) = A(b,s)\sigma_{jet}(s). \tag{20}$$

The QCD jet cross section drives the rise due to the increase with energy of the number of partonic collisions. It is calculated from the usual perturbative QCD expression, with DGLAP evoluted parton densities and perturbative partonic differential cross-sections. In Eq. (20), $A(b,s)$ is the overlap function which depends on the (energy dependent) spatial distribution of partons inside the colliding hadrons, averaged over the densities [1, 32]. Before discussing this function we shall examine the energy behaviour of the mini-jet cross-sections.

In the $\sqrt{s} >> p_{tmin}$ limit, the major contribution to the mini-jet cross-sections comes from collisions of gluons carrying small momentum fractions $x_{1,2} << 1$, a region where the relevant PDFs behave approximately like powers of the momentum fraction x^{-J} with $J \sim 1.3$ [33]. This leads to the asymptotic high-energy expression for σ_{jet}

$$\sigma_{jet} \propto \frac{1}{p_{t\,\min}^2}\left[\frac{s}{4p_{t\,\min}^2}\right]^{J-1} \tag{21}$$

where the dominant term is a power of s. Fits to the mini-jet cross-sections, obtained with different PDF sets [34] confirm the value $\varepsilon \equiv J-1 \sim 0.3$.

Such energy behaviour as in Eq. (21) is at odds with the gentle rise of the total pp and $p\bar{p}$ cross-sections at very high energy, described rather as $\ln s, \ln^2 s$ [11] or $s^{0.08}$ power [2]. However, we shall see that a proper implementation of other QCD processes can modify this strong rise. To do so we now examine the energy behaviour of the next component of our BN model, the impact parameter distribution.

We have identified soft gluon k_t-emission from the colliding partons as the physical effect responsible for the attenuation of the rise of the total cross section. These soft emissions break collinearity between the colliding partons, diminishing the efficiency of the scattering process. Their number increases with energy and thus their contribution remains important, even at very high energy, influencing matter distribution inside the hadrons, hence changing the overlap function, which is proposed to be the Fourier transform of the previous expression for the soft gluon transverse momentum resummed distribution, i.e,. we put

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \, e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b} \; e^{-h(b,q_{max})}} = A_0(s) e^{-h(b,q_{max})} \tag{22}$$

The integral in $h(b, q_{max})$ is performed up to a value q_{max}, which is linked to the maximum transverse momentum allowed by kinematics of single gluon emission [29]. In principle, this parameter and the overlap function should be calculated for each partonic sub-process, but in the partial factorization of Eq.(20) we use an average value of q_{max} obtained by considering all the sub-processes that can happen for a given energy of the main hadronic process[28]. The energy parameter q_{max} is of the order of magnitude of p_{tmin}. For present low$-x$ behaviour of the PDFs, in the high energy limit, q_{max} is a slowly varying function of s, starting as $\ln s$, with a limiting behaviour which depends on the densities [35]. From Eqs. (16) and (21) one can estimate the very large s-limit

$$n_{hard}(b,s) = A_{BN}(b,s)\sigma_{jet}(s, p_{tmin}) \sim A_0(s) e^{-h(b,q_{max})} \sigma_1 (\frac{s}{s_0})^\varepsilon \tag{23}$$

and, from this, using the very large b-limit,

$$n_{hard}(b,s) \sim A_0(s)\sigma_1 e^{-(b\bar{\Lambda})^{2p}} (\frac{s}{s_0})^\varepsilon \tag{24}$$

with $A_0(s) \propto \Lambda^2$ and with a logarithmic dependence on q_{max}, i.e. a very slowly varying function of s. We also have

$$\bar{\Lambda} \equiv \bar{\Lambda}(b,s) = \Lambda \{ \frac{c_F \bar{b}}{4\pi(1-p)} [\ln(2q_{max}(s)b) + \frac{1}{1-p}] \}^{1/2p} \tag{25}$$

In the next section, we shall see how the two critical exponents of our model, namely the power ε with which the mini-jet cross-section increases with energy and the parameter p dictating the IR behaviour of the QCD coupling constant, combine to obtain a rise of the total cross-section in agreement with the $\ln^2 s$ limitation imposed by the Froissart bound.

5. Ultra soft gluons in the IR limit and the asymptotic limit of the total cross-section: the Froissart bound

In this section we consider the very large s-limit of the total cross-section, in the approximation that all constant (or decreasing) terms in the eikonal function can be neglected, thus studying only the QCD effects from mini-jets and soft k_t-resummation. We find a link between the infrared behaviour of the ultra soft gluons and the asymptotic Froissart-like behaviour of the total cross-section and discuss it.

Let us consider the total cross-section in the eikonal representation at very large asymptotic energies. At such large energies that $n_{NP} << n_{hard}$, the total cross-section in our model [1] reads

$$\sigma_T(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-n_{hard}(b,s)/2}] \tag{26}$$

We consider the asymptotic expression for σ_{jet} at high energies, which grows like a power of s, and $A_{BN}(b,s)$, which was obtained through soft gluon resummation, and which decreases in b-space at least like an exponential $(1 < 2p < 2)$. In such large-b, large-s limit, we can write

$$n_{hard} = 2C(s) e^{-(b\bar{\Lambda})^{2p}} \tag{27}$$

where $2C(s) = A_0(s)\sigma_1(s/s_0)^\varepsilon$. The resulting expression for σ_T is

$$\sigma_T(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}] \tag{28}$$

With the variable transformation $u = (\bar{\Lambda}b)^{2p}$, and neglecting the logarithmic b-dependence in $\bar{\Lambda}$ by putting $b = 1/\Lambda$, Eq. (28) becomes

$$\sigma_T(s) \approx \frac{2\pi}{p}\frac{1}{\bar{\Lambda}^2} \int_0^\infty du u^{1/p-1} [1 - e^{-C(s)e^{-u}}] \tag{29}$$

Notice that, as $s \to \infty$, $C(s)$ also grows indefinitely as a power law. This means that the quantity between square brackets $I(u,s) = 1 - e^{-C(s)e^{-u}}$ has the limits $I(u,s) \to 1$ at $u = 0$ and $I(u,s) \to 0$ as $u = \infty$. Calling u_0 the value at which $I(u_0,s) = 1/2$ we then put $I(u,s) \approx 1$ and integrate only up to u_0. Thus

$$\bar{\Lambda}^2 \sigma_T(s) \approx (\frac{2\pi}{p}) \int_0^{u_0} du u^{\frac{1-p}{p}} = 2\pi u_0^{1/p} \tag{30}$$

and since, by construction

$$u_0 = \ln[\frac{C(s)}{\ln 2}] \approx \varepsilon \ln s \tag{31}$$

we finally obtain

$$\sigma_T \approx \frac{2\pi}{\bar{\Lambda}^2}[\varepsilon \ln \frac{s}{s_0}]^{1/p} \tag{32}$$

to leading terms in $\ln s$. We therefore derive the asymptotic energy dependence

$$\sigma_T \rightarrow [\varepsilon \ln(s)]^{(1/p)} \tag{33}$$

apart from a possible very slow s-dependence from $\bar{\Lambda}^2$. The same result is also obtained using the saddle point method.

This indicates that the Froissart bound is saturated if $p = 1/2$. We shall now show that, in our model, analyticity demands $p > 1/2$ and thus that, no matter how fast the mini-jet cross-section may grow with energy, the Froissart bound is always satisfied.

The requirement that $p > 1/2$ follows from analyticity arguments in the complex z_s-plane, where $z_s = \cos\theta_s = 1 + 2t/(s-4m^2) \rightarrow 1 + 2t/s$ is the cosine of the s-channel scattering angle for the equal mass case. Basically, this limitation comes from asking that for large values of the impact parameter space b, the eikonal be such to decrease at least like an exponential, i.e. to have at least

$$\mathcal{I}m\chi(b,s) \rightarrow e^{-b\sqrt{t_0}} \tag{34}$$

where $t_0 > 0$ is the boundary of the Lehmann ellipse on the real axis in the z_s plane. To see the argument, consider the elastic scattering amplitude for a process $a + b \rightarrow a + b$, normalized in such a way that

$$\sigma_{tot} = 2\pi \mathcal{I}m F(s, t = 0) \tag{35}$$

with

$$F(s,t) = i \int d^2\mathbf{b}[1 - e^{i\chi(b,s)}] J_0(b\sqrt{-t}) \tag{36}$$

While in the s-channel physical region, and for equal mass particles, $s \geq 4m^2$ and $t \leq 0$, for the Lehmann ellipse $F(s,t)$ is analytic for $t \leq t_0 \leq \mu^2$ where μ is some hadronic mass (e.g. twice the pion mass, as the smallest mass exchanged in the t-channel). The actual value is unimportant, what is needed is $t_0 > 0$. Then, for $0 < t \leq t_0$ the argument of J_0 becomes imaginary so that

$$F(s, t > 0) = i \int d^2\mathbf{b}[1 - e^{i\chi(b,s)}] I_0(b\sqrt{t}) \tag{37}$$

where I_0 is the Bessel function of the second kind. The asymptotic behaviour of the Bessel functions is such that $I_0(y)$ for y real, grows exponentially as y

becomes large. The integral at the r.h.s. of the above equation has to exist up to values of $t = t_0$ and for fixed $(0 \leq t \leq t_0)$. Since, for large b-values,

$$I_0(b\sqrt{t}) \simeq e^{b\sqrt{t}}, \tag{38}$$

for the integrand to be finite the imaginary part of the eikonal function, $\mathcal{I}m\chi(b,s)$ must go to zero at least like an exponential, i.e. at least

$$\mathcal{I}m\chi(b,s) \Rightarrow e^{-b\sqrt{t_0}} \tag{39}$$

Now, let us return to the expression for $\mathcal{I}m\chi(b,s)$ from the BN model, where $\mathcal{I}m\chi(b,s) \simeq e^{-h(b,s)}$ up to exponential accuracy. Using the large-b behaviour of the function $h(b,s)$, derived in previous papers and reproduced in the previous section, we see that

$$\mathcal{I}m\chi(b,s) \simeq e^{-h(b,s)} \simeq e^{-(\frac{b}{b_0})^{2p}} \tag{40}$$

whence follows that $p > 1/2$.

6. About scales and parameters in the BN model

The model we have described contains different scales. As mentioned earlier, in this paper we only study the rising part of the cross-section, whose behaviour is proposed to come from processes for which the outgoing partons have transverse momentum $p_t \geq p_{tmin} \simeq \mathcal{O}(1\ GeV)$. In our model, p_{tmin} is the scale which separates perturbative scattering processes from everything else. Soft gluon emission introduces two more scales, namely Λ and q_{max}. The latter is the maximum transverse momentum in the integral for soft gluon emission, it is of order p_{tmin} and plays the role of the energy scale E which appeared in QED radiative correction factors. Thus soft gluons satisfy the condition

$$k_t \leq q_{max} \simeq \mathcal{O}(p_{tmin}), \tag{41}$$

since most of the parton-parton cross-section is peaked at $p_t = p_{tmin}$. The next scale $\Lambda \simeq \mathcal{O}(\Lambda_{QCD})$ separates the region of ultra soft gluons from the rest. This region was originally neglected, on the basis that gluons with $|k_\perp| \ll \Lambda$ would see the hadron as a point-like object [36] and such emissions would have a small probability, because of colour screening. This argument is appealing, and similar to the one mentioned in Sect. 2, but in our opinion, there is no compelling theoretical reason to assume that ultrasoft gluon emission in high energy reactions has low probability. This argument could be applied to an isolated hadron, but not to high energy hadronic scattering described through the scattering of partons, where soft gluon emission is stimulated by QCD interactions. It is through this interaction that we can expect the transition between hadrons and quarks to arise. A singularity in the infrared region would indeed provide a cut-off to separate quarks from hadrons and lead to such transition. This is the rationale behind going into the zero momentum region, and enter it with a singular confining coupling between ultra-soft gluons and the quark current. In our model, these ultra soft gluons are important for the extremely large impact parameter values, which enter eikonal formulations of the total cross-section at very large energy. For processes where such large-b values do not play a role, this region may be irrelevant, though.

The ultra soft gluon distribution which we have introduced depends on the parameter p which regulates the infrared region of the soft gluon integral of Eq. (7). In [1], we have used the value $p = 3/4$ but this is a phenomenological value which was obtained after performing various averages over the PDFs. A determination of the actual value of p is beyond the scope of this paper, except for the fact that $p < 1$ for the soft gluon integral to converge and $p > 1/2$ for analyticity of the scattering amplitude.

One can evaluate the coefficient of the $(\ln s)^{1/p}$ term in Eq. (33) as given by

$$C = \frac{2\pi}{m_\pi^2}(\frac{m_\pi}{\Lambda})^2(\frac{27(1-p)^2\varepsilon}{4[1+(1-p)\ln(2q_{max}/\Lambda)]})^{1/p} \tag{42}$$

for $N_f = 3$, as appropriate for the total cross-section limit. In the approximation in which the q_{max} term is neglected, Eq. (42) gives $\sigma_T \approx \pi/m_\pi^2 \ln^2 s$ for $p = 1/2$, $\Lambda = 100\ MeV$ and $\varepsilon = 0.3$. However, q_{max}, which provides an extra dynamical scale, cannot, in general, be neglected: using Eq. (42) with $q_{max} \simeq 1\ GeV$, the above equation gives $C \simeq \mathcal{O}(0.1)\pi/m_\pi^2$, namely a constant which is one order of magnitude smaller than in the case of the actual Froissart bound [4].

We note that in the eikonal model as used here, one can derive the limit for the inelastic cross-section [37] following the same steps as above obtaining a constant reduced by a factor 2, as in a black disk model.

Conclusions

Using an eikonal mini-jet model for the total cross-section we have shown how soft gluon k_t-resummation in the IR region can reduce the strong power-like rise due to the minijet cross-section. We have found that this model will always satisfy the bound $\sigma_{total} \leq \ln^2 s$ provided that the infrared behaviour of α_s reflect a rising one gluon exchange potential: for a potential rising like r^{2p-1} the total cross-section is limited by an asymptotic behaviour $\simeq (\ln\ s)^{1/p}$, with $1/2 < p < 1$. This establishes the connection, in our BN model, between confinement and the satisfaction of limitations imposed by the Froissart bound.

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